Some Examples of Automorphism in a Limited Group

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Abstract: Some examples of automorphism in a group with limited liability are considered in the article, a brief analysis is done.

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We will first describe the concepts related to this topic.

If X and U are optional divided into sets, each $x \in$ X If an element with a definite value $y \in$ U is matched for an element, then X is given φa reflection (and is usually denoted as φ: X $\to$ U) and the element *u is* called the *inverse of the* element *x* , and *x* is *the original of the* element *u* , and this is the reflection. is written as u = φ(x) .

Definition. φ: X $\to$ U reflection X set to U set *in zaro one precious reflection* is called if each one in U *element* $\in$ X is the only one in fact owner if any.

Definition. Optional _ from the elements in a structured G set detected • binary operation for q in the house conditions if _ $\langle G, \bullet \rangle$ couple or G *group* is called:

1) *associative* : $\forall a, b , c \in$ G for ($a \bullet in$) $\bullet c = a \bullet$ ($in \bullet c$)

2) So it is in G *e* element is present then for it $e \bullet a = a \bullet e = a$ equality optional $a \in$ G for the element if; such *e* element G grouping *unit (neutral) element* is called.

3) $\forall$ for $a \in$ G element G has such an element a^{-1} then for him $a \bullet a^{-1} = a^{-1} \bullet a = e$ equality if. Such element *a* element is called the inverse element.

For example. All in all numbers package Z is the usual defined in it form a group in relation to the tumor action. Adding integers has an associative property It is known from the laws of arithmetic q, that the number zero is the function of the neutral element in this set does ($\forall n \in$ Z for $0 + n = n + 0 = n$). In addition, for each *n* element, there will be an opposite element - *n* (*n* + (- *n*) = (- *n*) + *n* = 0).

this group *all of numbers to add relatively group* and is denoted by (Z , +) .

The basic set is limited from the elements formed found groups limited groups is called in the group elements sony grouping procedure is called An infinite number of elements A group that includes itself is called an infinite group .

Definition. G $_1$ and G $_{2\ are}$ given in 2 groups get and G $_1$ group elements to G $_2$ group elements hand washer so *reciprocal a q valuable φ reflection* let G $_1$ basic binary application G$_2$ basic binary instead let, that is, if G is in group $_1$φ(*a*) = *a* ′, φ(*v*) = *v* ′, φ(*c*) = c′ and $a * v = c$ if it is h ahead of G $_2$ in a group $a' \circ v' = c'$ get _ In that case φ — G $_1$ G $_2$ hand washer *isomorphism* isomorphism is established groups *reciprocal isomorphic groups* is called.

Reciprocal a q valuable reflection -φ being an isomorphism condition again q at home write you can: $\forall a, v \in$ For G $_1$ φ($a * v$) = φ(*a*) $\circ$ φ(*v*), this here $a * v$ the product is obtained at G $_1$, φ(*a*)

∘ φ(*c*) multiplication is obtained in G $_2$.

Definition. G group, *g* - his elements get _ G *groups self to himself conductive* φ$_g$ *reflection* option q at home: φ$_g$ (*h*) = *ghg* $^{-1}$ (where *h* grouping optional elements).

This is it from the reflection g element structured G group *internal automorphism* is called.

Grouping internal automorphism group self to himself conductive isomorphism be ordinary _

Indeed, *ghg* $^{-1}$ = *h* $_1$ is only and only if *h* = *g* $^{-1}$ *h* $_1$ *g* . Therefore, as a φresult of the reflection of $_g$, the origin of the element *h* $_1$ exists and becomes unique. It follows that the reflection φ$_g$ (*h*) = *ghg* $^{-1}$ is a self-conducting reciprocal one-valued reflection of the group. In addition,

φ$_g$ (*h* $_1$ *h* $_2$) = *g* (*h* $_1$ *h* $_2$) *g* $^{-1}$ = *gh* $_1$ (*g* $^{-1}$ *g*) *h* $_2$ *g* $^{-1}$ = (*gh* $_1$ *g* $^{-1}$) (*gh* $_2$ *g* $^{-1}$) = = φ$_g$ (*x* $_1$) φ$_g$ (*x* $_2$). Shuning uchun φ- isomorphism of white.

These concepts are important concepts of abstract algebra. Delivering them to students poses unique challenges. With this in mind, the use of regular polygon substitution groups in the plane in explaining internal automorphisms in finite groups gives students some convenience.

For example. Regular triangle symmetry group Let's look at q (Figure 1). Of a triangle with ends A, B, C. if we define it, it is h ahead this grouping elements three h of the letters A, B, C. place exchange a q value. For example , from end A to VS lowered height relative symmetry $\begin{pmatrix} ABC \\ ACB \end{pmatrix}$ in view is written .
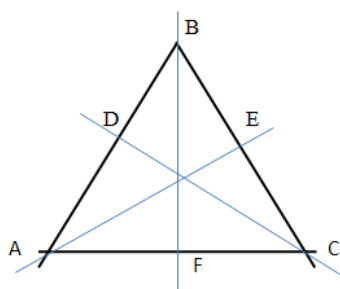
1 - picture

Triangle symmetry in the group two elements multiply for first place exchange from the second then do need will be . This is it h in front , triangle symmetry group and three h of the letters A, B , C. place exchanges group in the middle installed isomorphism has we will This isomorphism is one q valuable that it is not clear emphasize : it is a triangle exactly q which end with A , which end q with V and C. to determine. Triangle tips designation self h am A, V, S h of the letters place exchange as possible . It 's a corner symmetry in the group h each element of the letters A, B, S h place exchange in view new to determine has will be .

Triangle symmetry in the group all possible divided inside automorphisms as a result to heights relative symmetries k aysi to the elements Let 's take a look at the issue.

Symmetries with respect to heights:

$$c = \begin{pmatrix} ABC \\ ACB \end{pmatrix}, \quad d = \begin{pmatrix} ABC \\ CBA \end{pmatrix}, \quad f = \begin{pmatrix} ABC \\ BAC \end{pmatrix},$$

1) φ$_a$ (*c*) = *aca* $^{-1}$ = (BCA) (ACB) (CAB) = (CBA) (CAB) = (BAC) = *f*

2) φ$_b$ (*c*) = *bcb* $^{-1}$ = (CAB) (ACB) (BCA) = (BAC) (BCA) = (CBA) = *d*

3) φ$_c$ (*c*) = *c c c* $^{-1}$ = *a c* $^{-1}$ = *a c* = *c*

4) φ$_d$ (*c*) = *d c d* $^{-1}$ = *b d* = *f*

5) $\varphi_f(c) = f\,c\,f^{-1} = a\,f = d$

6) $\varphi_a(d) = a\,d\,a^{-1} = c\,b = f$

7) $\varphi_b(d) = b\,d\,b^{-1} = f\,a = c$

8) $\varphi_c(d) = c\,d\,c^{-1} = a\,c = f$

9) $\varphi_d(d) = d\,d\,d^{-1} = b\,d = d$

10) $\varphi_f(c) = f\,d\,f^{-1} = c$

11) $\varphi_a(f) = a\,f\,a^{-1} = c$

12) $\varphi_b(f) = b\,f\,b^{-1} = d$

13) $\varphi_c(f) = c\,f\,c^{-1} = d$

14) $\varphi_d(f) = d\,f\,d^{-1} = c$

15) $\varphi_f(f) = fff^{-1} = f$

Hence, as a result of all possible internal automorphisms in the group of triangular symmetries, the symmetries with respect to the heights shift to all the height symmetries.

let us determine which elements the conversion of a triangle to 120^0 results in as a result of possible internal automorphisms in the group of triangle symmetry .

$$a = \begin{pmatrix} ABC \\ BCA \end{pmatrix},$$

1) $\varphi_a(a) = a\,a\,a^{-1} = b\,b = a$

2) $\varphi_b(a) = b\,a\,b^{-1} = e\,a = a$

3) $\varphi_c(a) = c\,a\,c^{-1} = d\,c = b$

4) $\varphi_d(a) = d\,a\,d^{-1} = b$

5) $\varphi_f(a) = faf^{-1} = b$

6) $\varphi_e(a) = eae^{-1} = a$

conversion of a triangle to 120^0 as a result of possible internal automorphisms in the group of triangle symmetry Answer : a - 120^0 and b - 240^0 to convert passes .

The same problem can be solved for other regular polygons. For example, when this problem is considered in the group of tetrahedral symmetry, if all its elements are divided into the following classes: 1) e ; 2) all rotations around the heights other than e ; 3) 180 ° rotation around the arrow passing through the opposite ^{edge} ; 4) symmetry with respect to the plane passing through any edge and the edge opposite it; 5) all rotations resulting from the cyclic displacement of the ends (for example $\begin{pmatrix} ABCD \\ BCDA \end{pmatrix}$), in which case the two elements in the group of tetrahedral symmetry can pass to each other as a result of internal automorphism, and only when they can be summed up in one class.

It should be noted that in an arbitrary internal automorphism of a group (such as an arbitrary isomorphism), each group of its parts, in general, passes into a completely different group (for example, the symmetry of a triangle relative to the height of one side passes to the symmetry of another height). However, some "special symmetric" subgroups remain in place as a result of internal automorphism (for example, in the triangular symmetry subgroup of a triangle symmetry group).

In conclusion, given the specific complexity of the concepts of abstract algebra, their delivery to students, the formation of imagination and skills related to the practical application of these concepts requires a specific approach, the choice of practical problems is important.

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